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328351(14)

B. E. (Third Semester) Examination, April-May 2020

NOV-DEC 2020

(New Scheme)

(ET & T Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : *Attempt all questions. Part (a) of each question is compulsory and carries 2 marks. Solve any **two** parts from (b), (c) and (d) carrying 7 marks each.*

Unit-I

1. (a) Write the conditions for the existence of Laplace transform. 2

(b) Find the Laplace transform of

$$\frac{1-\cos t}{t^2}$$ 7

(c) Apply convolution theorem to evaluate :

$$L^{-1}\left\{\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right\}$$ 7

(d) Solve the equation by the transform method :

$$ty''+(1-2t)y'-2y=0$$

when $y(0)=1, y'(0)=2$. 7

Unit-II

2. (a) State residue theorem. 2

(b) If $f(z)$ is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$$

(c) Expand

$$f(z)=\frac{1}{(z-1)(z-2)}$$

in Laurent series valid for :

(i) $1<|z|<2$

(ii) $|z|>2$

(iii) $|z|<1$ 7

(d) By integrating around a unit circle, evaluate :

$$\int_0^{2\pi}\frac{\cos 2\theta \, d\theta}{5+4\cos\theta}$$ 7

Unit-III

3. (a) Write about Karl-Pearson's coefficient of correlation. 2

(b) If θ is the acute angle between the two regression lines in the case of two variables x and y.

Show that :

$$\tan\theta = \frac{1-r^2}{r}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$

Explain the significance of the formula when $r = 0$ and $r = \pm 1$. 7

(c) Find the coefficient of correlation and regression lines to the following data :

X	:	5	7	8	10	11	13	16
Y	:	33	30	28	20	18	16	9

(d) Two judges in a beauty contest rank the ten competitors in the following order :

6	4	3	1	2	7	9	8	10	5
4	1	6	7	5	8	10	9	3	2

Do the two judges appear agree in their standards? 7

Unit-IV

4. (a) Write Rodrigue formula for $P_n(x)$. 2

(b) Solve in series the equation :

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$$ 7

(c) Show that :

(i) $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$ 3

(ii) $J_{1/2}(x) = \sqrt{\frac{2}{(\pi x)}}\sin x$ 4

(d) Prove that :

$$\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = \begin{cases} 0 & : \quad m \neq n \\ \dfrac{2}{2n+1} & : \quad m = n \end{cases}$$ 7

Unit-V

5. (a) Form the partial differential equation be eliminating the arbitrary constant :

$$z = ax + by + a^2 + b^2$$ 2

(b) Solve :

$$(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$$ 7

(c) Solve :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x + y)$$ 7

(d) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set virbrating by giving each point a velocity $v_0 \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x, t)$. 7